Answer on Question #69038 – Math – Real Analysis

Question

Check whether the following functions are continuous or not at x = 0. Also, find the nature of discontinuity at that point, if it exists.
(i) f(x)= {[(2-x)^(1/2) - (2+x)^(1/2)] / [x] , x≠0 ; 1/√2 , x=0
(ii) f(x)= { x^(2)+1/3, x≤0 ; -[x^(3)+1/3], x>0

Solution

(i) $f(x) = \begin{cases} \frac{\sqrt{2-x}-\sqrt{2+x}}{x}, & x \neq 0 \\ \frac{1}{\sqrt{2}}, & x = 0 \end{cases}$

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = \lim_{x\to 0} f(x) = -\frac{1}{\sqrt{2}}.$

$f(0) = \frac{1}{\sqrt{2}}.$

Thus, $f(x)$ has a removable discontinuity at $x = 0$.

(ii) $f(x) = \begin{cases} x^2 + \frac{1}{3} & x \leq 0 \\ -x^3 - \frac{1}{3}, & x > 0 \end{cases}$

$\lim_{x\to 0^-} f(x) = \frac{1}{3}, \lim_{x\to 0^+} f(x) = -\frac{1}{3}.$

Thus, $f(x)$ has a jump discontinuity at $x = 0$ (discontinuity of the first kind).

Answer: (i) discontinuous; a removable discontinuity; **(ii)** discontinuous, a jump discontinuity.