Answer on Question #68955 – Math – Linear Algebra

Question

Let $V = \mathbb{R}^2$. Define addition + on V by

$$(x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$$

and scalar multiplication $\cdot$ by

$$r \cdot (a, b) = (ra, 0).$$

Check whether V satisfies all the conditions for it to be a vector space over $\mathbb{R}$ with respect to these operations.

Solution

V is not a vector space over $\mathbb{R}$ with respect to these operations. The axiom of identity element of scalar multiplication is not fulfilled.

In order to be a vector space, V has to fulfill the axiom:

$$1 \cdot (a, b) = (a, b)$$

for arbitrary $(a, b) \in V$, where 1 denotes the multiplicative identity in $\mathbb{R}$.

But for example, for $(a, b) \in V$ with $b \neq 0$ for arbitrary $r \in \mathbb{R}$ we have

$$r \cdot (a, b) = (ra, 0) \neq (a, b).$$

Therefore, V is not a vector space over $\mathbb{R}$ with respect to these operations.

Answer: V is not a vector space over $\mathbb{R}$ with respect to these operations.