Answer on Question #68893 – Math – Calculus

Question

Find the nth derivative of a^{mx}

Solution

$u(x) = a^{mx} = e^{mx\cdot\ln(a)}$,

$$\frac{d\left(a^{mx}\right)}{dx} = \frac{du}{dx} = m\cdot\ln(a)\cdot e^{mx\cdot\ln(a)} = m\cdot\ln(a)\cdot a^{mx} = m\cdot\ln(a)\cdot u(x),$$

$$\frac{d^2u}{dx^2} = \frac{d}{dx}\left(\frac{du}{dx}\right) = \frac{d}{dx}\left(m\cdot\ln(a)\cdot u(x)\right) = m\cdot\ln(a)\frac{du}{dx} = m\cdot\ln(a)\cdot m\cdot\ln(a)\cdot u(x) = m^2\left(\ln(a)\right)^2\cdot u(x)$$

. . .

$$\frac{d^n\left(a^{mx}\right)}{dx^n} = \frac{d^nu}{dx^n} = m^n\left(\ln(a)\right)^n\cdot u(x) = m^n\left(\ln(a)\right)^n a^{mx}.$$

Answer: $\frac{d^nu}{dx^n} = m^n\left(\ln(a)\right)^n a^{mx}$.