

Question #68628, Math / Other

Find the radius of the sphere which passes through the points $(0,0,0)$, $(1,0,0)$, $(0,1,0)$ and $(0,0,1)$

Answer.

Equation of the sphere: $(x - a)^2 + (y - b)^2 + (z - c)^2 = r^2$.

Points $(0,0,0)$, $(1,0,0)$, $(0,1,0)$ and $(0,0,1)$ lie on the sphere.

$$\text{So } \begin{cases} a^2 + b^2 + c^2 = r^2 \\ (1 - a)^2 + b^2 + c^2 = r^2 \\ a^2 + (1 - b)^2 + c^2 = r^2 \\ a^2 + b^2 + (1 - c)^2 = r^2 \end{cases} \rightarrow \begin{cases} a^2 + b^2 + c^2 = r^2 \\ 1 - 2a + a^2 + b^2 + c^2 = r^2 \\ a^2 + 1 - 2b + b^2 + c^2 = r^2 \\ a^2 + b^2 + 1 - 2c + c^2 = r^2 \end{cases} \rightarrow$$
$$\rightarrow \begin{cases} 1 - 2a = 0 \\ 1 - 2b = 0 \\ 1 - 2c = 0 \end{cases} \rightarrow a = b = c = \frac{1}{2}.$$

$$\text{Thus, } r^2 = a^2 + b^2 + c^2 = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4} \rightarrow r = \frac{\sqrt{3}}{2}.$$