Find the area of the parallelogram with sides $a = i - 2j + k$ and $b = 2i + j + k$

Solution
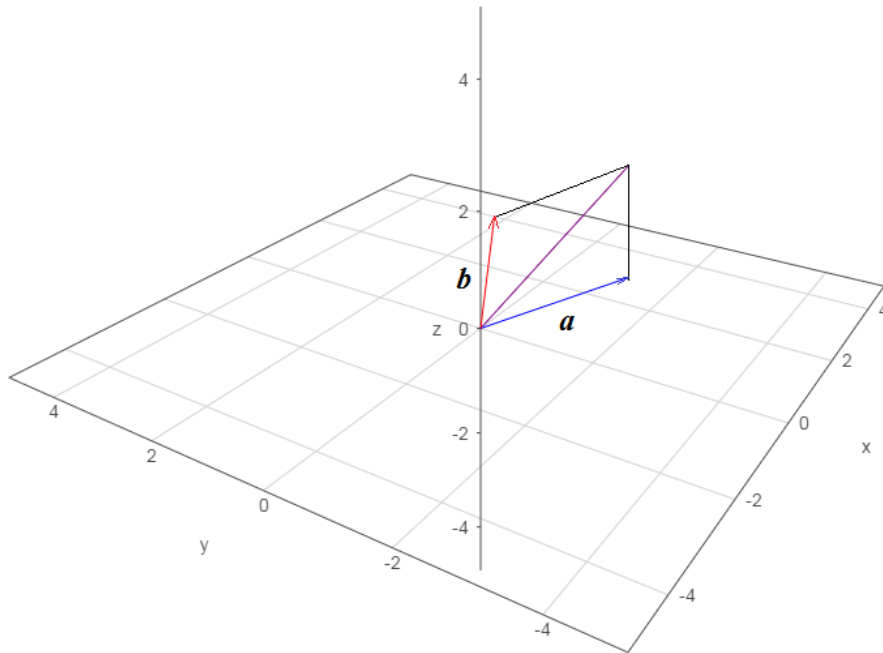
The area of the parallelogram will be the length of the cross product of adjacent sides. We have

$$a \times b = \begin{vmatrix} i & j & k \\ 1 & -2 & 1 \\ 2 & 1 & 1 \end{vmatrix} =$$

$$= (-1)^{1+1} i \begin{vmatrix} -2 & 1 \\ 1 & 1 \end{vmatrix} + (-1)^{1+2} j \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} + (-1)^{1+3} k \begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix} =$$

$$= i(-2-1) - j(1-2) + k(1-(-4)) = -3i + j + 5k$$

Thus, the area of the parallelogram is

$$|-3i + j + 5k| = \sqrt{(-3)^2 + (1)^2 + (5)^2} = \sqrt{35} \ (units^2)$$

Answer: $\sqrt{35} \ units^2$.