Answer on Question #68581, Math / Differential Equations

Solve the differential equation

$$(1 + yx)xdy + (1 - yx)ydx = 0$$

Solution

The trivial solutions

$$y(x) = 0 \text{ and } x = 0$$

$$(1 + yx)xdy + (1 - yx)ydx = 0$$
$$(1 + yx)x\frac{dy}{dx} + (1 - yx)y = 0$$
$$\frac{dy}{dx} = y'(x) = \frac{y(yx - 1)}{x(1 + yx)}$$

$$t(x) = xy, t'(x) = (xy)' = y + xy' = \frac{t}{x} + x\frac{y(yx - 1)}{x(1 + yx)} =$$
$$= \frac{t}{x} + \frac{t(t - 1)}{x(1 + t)} = \frac{t + t^2 + t^2 - t}{x(1 + t)} = \frac{2t^2}{x(1 + t)}$$
$$\frac{dt}{dx} = \frac{2t^2}{x(1 + t)}$$
$$dt = \frac{2t^2}{x(1 + t)}dx$$

This is a separable ODE.

$$\frac{1 + t}{2t^2}dt = \frac{dx}{x}$$

Integrate both sides

$$\int \frac{1 + t}{2t^2}dt = \int \frac{dx}{x}$$

$$\int \frac{1}{2t^2}dt + \int \frac{1}{2t}dt = \ln|x| - \frac{1}{2}\ln C$$
$$-\frac{1}{2t} + \frac{1}{2}\ln|t| = \ln|x| - \frac{1}{2}\ln C$$
$$\frac{1}{t} = \ln\left(\frac{C|t|}{x^2}\right)$$
$$\frac{1}{xy} = \ln\left(\frac{C|xy|}{x^2}\right)$$
$$xy\ln\left(C\left|\frac{y}{x}\right|\right) = 1$$

Answer: $x = 0;\ y(x) = 0;\ xy\ln\left(C\left|\frac{y}{x}\right|\right) = 1.$