Question #68547, Math / Statistics and Probability

A fruit processing unit produces canned fruit juice with a mean weight of 1 kg. and variance of 0.020 kg. The cans are packed in boxes each containing 100 cans. Assuming that the weights of individuals cans are statistically independent find the probability that the weight of a box is between 99 kg. and 101.5 kg.

Solution

A box of 100 cans of weight between 99 and 101.5 kg is a sample of size 100 with sample mean in the following range:

99 / 100 = 0.99 kg

101.5 / 100 = 1.015 kg

The sampling distribution:

$\mu_{\bar{x}} = \mu = 1;$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{\sqrt{0.02}}{10} = 0.014$$

$$z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}};$$

$$z_1 = \frac{0.99 - 1}{0.014} = -0.71;$$

$$z_2 = \frac{1.015 - 1}{0.014} = 1.06$$

$$p(z_1 < z < z_2) = p(z < z_2) - p(z < z_1)$$

The cumulative *p*-values for the given z-scores can be obtained from the standard normal table or calculated using the technology (MS Excel function NORM.S.DIST()).

$p(z < -0.71) = 0.2398;$

$p(z < 1.06) = 0.8556$

$$p(z_1 < z < z_2) = 0.8556 - 0.2398 = 0.6158$$

Answer: the probability that the weight of a box is between 99 kg. and 101.5 kg is 0.6158.