Answer on Question#68542 – Math – Statistics and Probability

Question. The manufacturer of a popular brand of TV knows from his past experience that the probability of his TV set failing to work properly during the warranty period is 0.04. Find the probability that in a sample of 25 sold TV sets selected at random, 6 or less TV sets will be failing to work properly during the warranty period. Use both the binomial and Poisson rule and compare the results.

Solution. Let us start from the finding of exact probability using the binomial rule. See https://en.wikipedia.org/wiki/Binomial_distribution In our case $n = 25, p = 0.04, q = 0.96$. Then the required probability is:

$$P\{6\ or\ less\} = P\{0\} + P\{1\} + P\{2\} + P\{3\} + P\{4\} + P\{5\} + P\{6\} =$$

$$= \binom{25}{0} \cdot 0.04^0 \cdot 0.96^{25} + \binom{25}{1} \cdot 0.04^1 \cdot 0.96^{24} + \binom{25}{2} \cdot 0.04^2 \cdot 0.96^{23} + \binom{25}{3} \cdot 0.04^3 \cdot 0.96^{22} +$$

$$+\binom{25}{4} \cdot 0.04^4 \cdot 0.96^{21} + \binom{25}{5} \cdot 0.04^5 \cdot 0.96^{20} + \binom{25}{6} \cdot 0.04^6 \cdot 0.96^{19} =$$

$$= \frac{25!}{0! \cdot 25!} \cdot 0.04^0 \cdot 0.96^{25} + \frac{25!}{1! \cdot 24!} \cdot 0.04^1 \cdot 0.96^{24} + \frac{25!}{2! \cdot 23!} \cdot 0.04^2 \cdot 0.96^{23} + \frac{25!}{3! \cdot 22!} \cdot 0.04^3 \cdot 0.96^{22} +$$

$$+ \frac{25!}{4! \cdot 21!} \cdot 0.04^4 \cdot 0.96^{21} + \frac{25!}{5! \cdot 20!} \cdot 0.04^5 \cdot 0.96^{20} + \frac{25!}{6! \cdot 19!} \cdot 0.04^6 \cdot 0.96^{19} \approx 0.99996$$

(see Appendix).

Now let us compute the required probability approximately with using the rule of Poisson

(see http://faculty.vassar.edu/lowry/poisson.html). We have: $np = 25 \cdot 0.04 = 1$. Then

$$P\{6\ or\ less\} = P\{0\} + P\{1\} + P\{2\} + P\{3\} + P\{4\} + P\{5\} + P\{6\} \approx$$

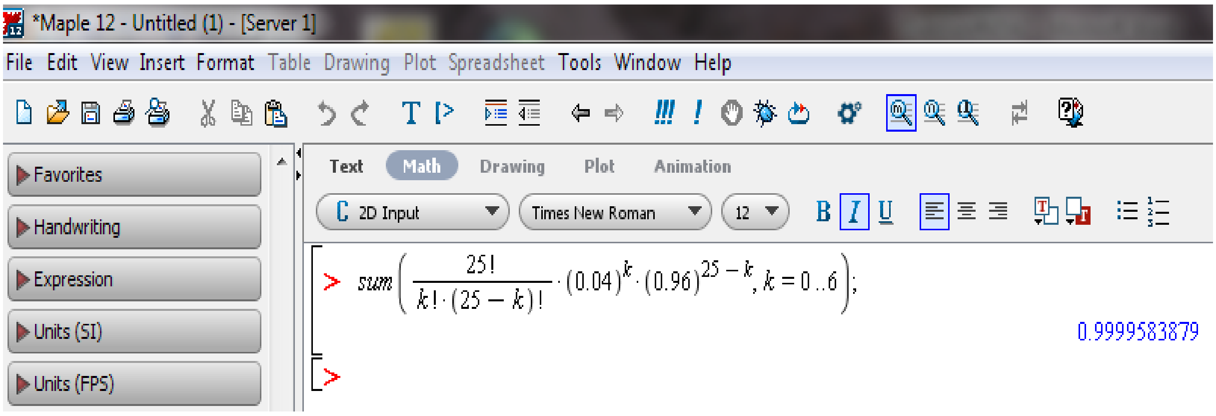
$$\approx \frac{1^0}{0!} e^{-1} + \frac{1^1}{1!} e^{-1} + \frac{1^2}{2!} e^{-1} + \frac{1^3}{3!} e^{-1} + \frac{1^4}{4!} e^{-1} + \frac{1^5}{5!} e^{-1} + \frac{1^6}{6!} e^{-1} =$$

$$= \frac{1}{e} \cdot \left(1 + 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + \frac{1}{120} + \frac{1}{720}\right) = \frac{1957}{720e} \approx 0.99992.$$

We see that the computational error is $|0.99996 - 0.99992| = 0.00004$ i.e. very small. But it is much easier to compute the required probability approximately using the rule of Poisson.

Answer. 0.99996.

Appendix


*Maple 12 - Untitled (1) - [Server 1]
File Edit View Insert Format Table Drawing Plot Spreadsheet Tools Window Help
Favorites
Handwriting
Expression
Units (SI)
Units (FPS)
Text Math Drawing Plot Animation
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12
B I U
> sum(25! / (k!·(25 − k)!) · (0.04)^k · (0.96)^(25 − k), k = 0 ..6);
0.9999583879
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