**Answer on Question #68294 – Math – Calculus**

## Question

Obtain the partial differential equation by eliminating the arbitrary constant from the relation

$$u = xy + y\sqrt{x^2 - a^2} + b$$

## Solution

$$u = xy + y\sqrt{x^2 - a^2} + b$$

$u_x = y + \frac{xy}{\sqrt{x^2-a^2}}$, $\quad u_y = x + \sqrt{x^2 - a^2}$ , $\quad x = u_y - \sqrt{x^2 - a^2}$

$u_{xx} = y\frac{\sqrt{x^2-a^2} - \frac{x^2}{\sqrt{x^2-a^2}}}{x^2-a^2}$ $\quad u_{yy} = 0$

$$u_{xy} = 1 + \frac{x}{\sqrt{x^2 - a^2}} = u_{yx} \quad \Rightarrow$$

$$\frac{x}{\sqrt{x^2 - a^2}} = u_{xy} - 1 , \quad \frac{u_y - \sqrt{x^2 - a^2}}{\sqrt{x^2 - a^2}} = u_{xy} - 1 , \frac{u_y}{\sqrt{x^2 - a^2}} = u_{xy}, \quad \sqrt{x^2 - a^2} = \frac{u_y}{u_{xy}}$$

$$x = u_y - \frac{u_y}{u_{xy}}$$

$$u_x = y\left(1 + \frac{x}{\sqrt{x^2 - a^2}}\right), u_x = y\left(1 + \frac{u_y - \frac{u_y}{u_{xy}}}{\frac{u_y}{u_{xy}}}\right), y = \frac{u_x}{1 + \frac{u_y(u_{xy} - 1)}{u_y}}, y = \frac{u_x}{u_{xy}}$$

$$u_{xx} = y\frac{\sqrt{x^2 - a^2} - \frac{x^2}{\sqrt{x^2 - a^2}}}{x^2 - a^2}$$

$$u_{xx} = y(\frac{1}{\sqrt{x^2 - a^2}} - \frac{x^2}{\sqrt{x^2 - a^2}(x^2 - a^2)})$$

$$u_{xx} = \frac{u_x}{u_{xy}}\left(\frac{u_{xy}}{u_y} - \frac{\left(u_y - \frac{u_y}{u_{xy}}\right)^2}{\left(\frac{u_y}{u_{xy}}\right)^3}\right)$$

$$u_{xx} = \frac{u_x}{u_{xy}}\left(\frac{u_{xy}}{u_y} - \frac{u_{xy}\left(u_{xy} - 1\right)^2}{u_y}\right)$$

$$u_{xx} = \frac{u_x}{u_y}\left(1 - \left(u_{xy} - 1\right)^2\right)$$

***Answer***: $u_{xx} = \frac{u_x}{u_y}\left(-\left(u_{xy}\right)^2 + 2u_{xy}\right).$