Find the unit outward drawn normal to the surface

$$(x-1)^2+y^2+(z+2)^2=9$$

at the point $(3,1,-4)$.

Solve:

We rewrite in the form

$$F(x,y,z)=(x-1)^2+y^2+(z+2)^2-9$$

Find the partial derivatives at the point

$$\frac{\partial F}{\partial x}=2(x-1)=2(3-1)=4$$

$$\frac{\partial F}{\partial y}=2y=2\cdot 1=2$$

$$\frac{\partial F}{\partial z}=2(z+2)=2(-4+2)=-4$$

Write the equation of the tangent plane

$$4(x-3)+2(y-1)-4(z+4)=0$$

Hence,

$$\overline{n}\|\overline{(4;2;-4)}.$$

Find a unit vector normal

$$\overline{n}=\frac{\overline{(4;2;-4)}}{\sqrt{4^2+2^2+(-4)^2}}=\overline{\frac{(4;2;-4)}{6}}=\overline{\left(\frac{2}{3};\frac{1}{3};-\frac{2}{3}\right)}.$$