Answer on Question #68040 – Math – Calculus

Question

Find the area bounded by the curve y=4x-x^2 and the line x=0 and y=4.

Solution

First, find the points of intersection of $y = 4x - x^2$ and $y = 4$:

$4x - x^2 = 4,$

$x^2 - 4x + 4 = 0,$

$(x - 2)^2 = 0,$

$x = 2,$

so

$y = 4x - x^2$ and $y = 4$ intersect at the point (2, 4).

The lines $x = 0$ and $y = 4$ intersect at the point (0, 4).

The curve $y = 4x - x^2$ and the line $x = 0$ intersect at the point (0, 0), because

$$y(0) = 4 \cdot 0 - 0^2 = 0.$$

The line $y = 4$ is higher than $y = 4x - x^2$, so the integrand will be as follows:

$$4 - 4x + x^2 .$$

So the area can be calculated as

$$\int_0^2 (4 - 4x + x^2)dx = (4x - 2x^2 + \frac{x^3}{3})\Big|_0^2 = 8 - 8 + \frac{8}{3} = \frac{8}{3}.$$

Answer: $\frac{8}{3}$.