

## Answer on Question #67547 – Math – Calculus

### Question

The projection map defined on  $(\mathbb{R}^2, d)$  where  $d$  is the usual metric is continuous. Is it true?

### Solution

Yes, it is true.

Consider the projection map  $pr_x: \mathbb{R}^2 \rightarrow \mathbb{R}$ ,  $pr_x(x, y) = x$  for each  $(x, y) \in \mathbb{R}^2$  (similarly for  $y$ ). We show that  $pr_x: \mathbb{R}^2 \rightarrow \mathbb{R}$  is continuous. Consider any point  $p_0 = (x_0, y_0) \in \mathbb{R}^2$ . Let  $\varepsilon > 0$ . Put  $\delta = \varepsilon$ . Then for each  $p = (x, y) \in \mathbb{R}^2$  such that  $d(p, p_0) < \delta$  we have that

$$|pr_x(p) - pr_x(p_0)| = |x - x_0| \leq d(p, p_0) < \delta = \varepsilon.$$

This means that the projection map  $pr_x: \mathbb{R}^2 \rightarrow \mathbb{R}$  is continuous at  $p_0$ . Then that projection map is continuous.