Answer on Question #67416 – Math – Calculus

A price-taking profit-maximising business, which operates at its optimal plant size, has a TC curve given by $TC = q^3 - 2q^2 + 2q + 0.576$ where TC is measured in 1000s of £ per week and q is the rate of output measured in 1000s of units per week.

Question

a) If the market price of output is $£1.00$ show that the firm will produce 1000 units per week and make a loss. Explain why the business is, at this price, indifferent between producing and closing down. Draw a diagram to show this.

Solution

MR - marginal revenue; MC - marginal cost; TR total revenue; p - price.

Since this is profit-maximising business, then

$$MR = MC$$

$$TR = pq$$

$$MR = \frac{d}{dq}(pq) = p$$

$$MC = \frac{d(TC)}{dq} = 3q^2 - 4q + 2$$

$$p = 3q^2 - 4q + 2$$

$$1 = 3q^2 - 4q + 2$$

$$q = \frac{4 \pm 2}{6}$$

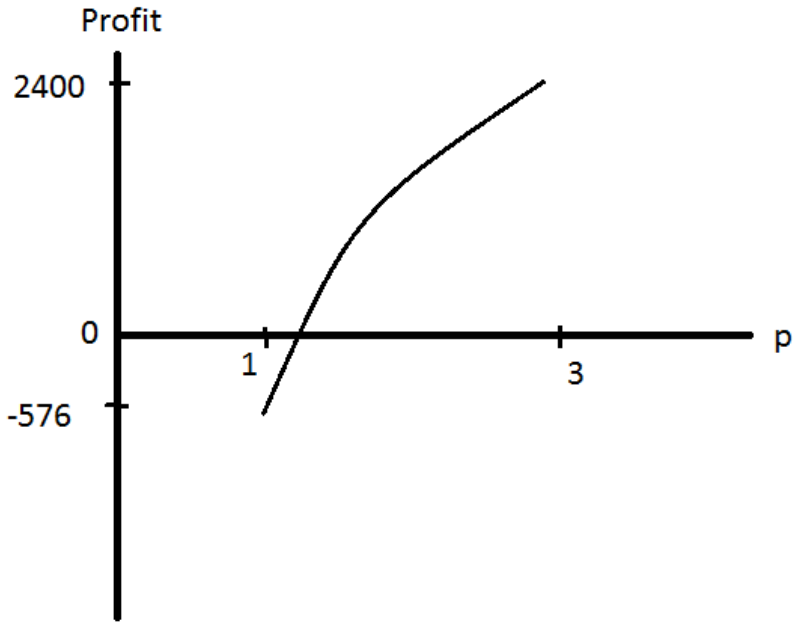
$$q_1 = 1 \ ; \ q_2 = \frac{1}{3}$$

Total Revenue:

$$TR = 1 \cdot q = 1000\,£\,per\ week$$

Then profit is

$$TR - TC = 1000 - (1 - 2 + 2 + 0.576) \cdot 1000 = -576\,£\,per\ week - the\ loss$$



Profit at this price is increasing, but on the other hand, it is negative. So the business is indifferent between producing and closing down.

Question

b) If the market price of output is £3.28 what will each firm produce and what profit is made?

Solution

$$p = 3q^2 - 4q + 2 = 3.28$$

$$q = \frac{4 \pm \sqrt{16 + 3 \cdot 1.28}}{6}$$

$$q = 1409\ units\ per\ week$$

Profit:

$$TR - TC = 3.28 \cdot 1409 - (1.409^3 - 2 \cdot 1.409^2 + 2 \cdot 1.409 + 0.576\ ) \cdot 1000$$
$$= 2400.82\ £\ per\ week$$

Question

c) Assuming that all firms in the industry have identical cost structures and that there is freedom of
entry, show that the long-run weekly rate of output will fall to 1200 units.

Solution

$$MC = \frac{TC}{q}$$

$$3q^2 - 4q + 2 = q^2 - 2q + 2 + \frac{0.576}{q}$$

$$2q^2 - 2q - \frac{0.576}{q} = 0$$

$$2q^3 - 2q^2 - 0.576 = 0$$

$$q = 1.2\ (1200\ units)$$

Question

d) What is the price of output when the industry is in equilibrium?

Solution

$$q = 1.2$$

$$p = 3q^2 - 4q + 2$$

$$p = 3 \cdot 1.2^2 - 4 \cdot 1.2 + 2 = 1.52 \text{ £}$$