Answer on Question #67085 – Math – Statistics and Probability

Question

If P(A) = 0.50, P(B) = 0. 40 and P(A ∪ B) = 0.70, find P(A | B) and P(A B), c ∪ where c A is the complement of A. State whether A and B are independent. Justify your answer.

Solution

By the addition law of probability,

$$P(\mathrm{A} \cup \mathrm{B}) = P(A) + P(B) - P(A \cap B).$$

Hence

$$P(\mathrm{A} \cap \mathrm{B}) = P(A) + P(B) - P(A \cup B) = 0.5 + 0.4 - 0.7 = 0.2.$$

By the definition of conditional probability,

$$P(\mathrm{A} \mid \mathrm{B}) = \frac{P(A \cap B)}{P(B)} = \frac{0.2}{0.4} = 0.5.$$

From the statement of the question it is not clear what should be found further. I shall find probabilities of some possible events.

By the complementary rule,

$$P((\mathrm{A} \cup \mathrm{B})^c) = 1 - P(A \cup B) = 1 - 0.7 = 0.3;$$

$$P((\mathrm{A} \cap \mathrm{B})^c) = 1 - P(A \cap B) = 1 - 0.2 = 0.8.$$

We recall that events A and B are independent if

$$P(\mathrm{A} \cap \mathrm{B}) = P(A)P(B).$$

In this case $P(\mathrm{A} \cap \mathrm{B}) = 0.2$ and $P(A)P(B) = 0.5 \cdot 0.4 = 0.2$.

Therefore, $P(\mathrm{A} \cap \mathrm{B}) = P(A)P(B)$, hence events A and B are independent.

Answer: $P(\mathrm{A} \mid \mathrm{B}) = 0.5$; $P((\mathrm{A} \cup \mathrm{B})^c) = 0.3$; $P((\mathrm{A} \cap \mathrm{B})^c) = 0.8$; events A and B are independent.