Answer on Question #67021 – Math – Statistics and Probability

Question

If P(A) = 0.50, P(B) = 0.40 and $P(A \cup B) = 0.70$, find $P(A \mid B)$ and $P(c(A \cup B))$, where $c(A \mid B)$ is the complement of A. State whether A and B are independent. Justify your answer.

Solution

By General Addition Rule for probabilities,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

Therefore,

$$P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.50 + 0.40 - 0.70 = 0.20.$$

By the definition of conditional probability,

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{0.20}{0.40} = 0.50.$$

By the complementary rule,

$$P(c(A \cup B)) = 1 - P(A \cup B) = 1 - 0.70 = 0.30.$$

For the given events A and B, $P(A \cap B) = 0.20$ and $P(A)P(B) = 0.50 \cdot 0.40 = 0.20$. Hence, $P(A \cap B) = P(A)P(B)$ and events A and B are independent.

Answer: $P(A \mid B) = 0.50$, $P(c(A \cup B)) = 0.30$, events A and B are independent.