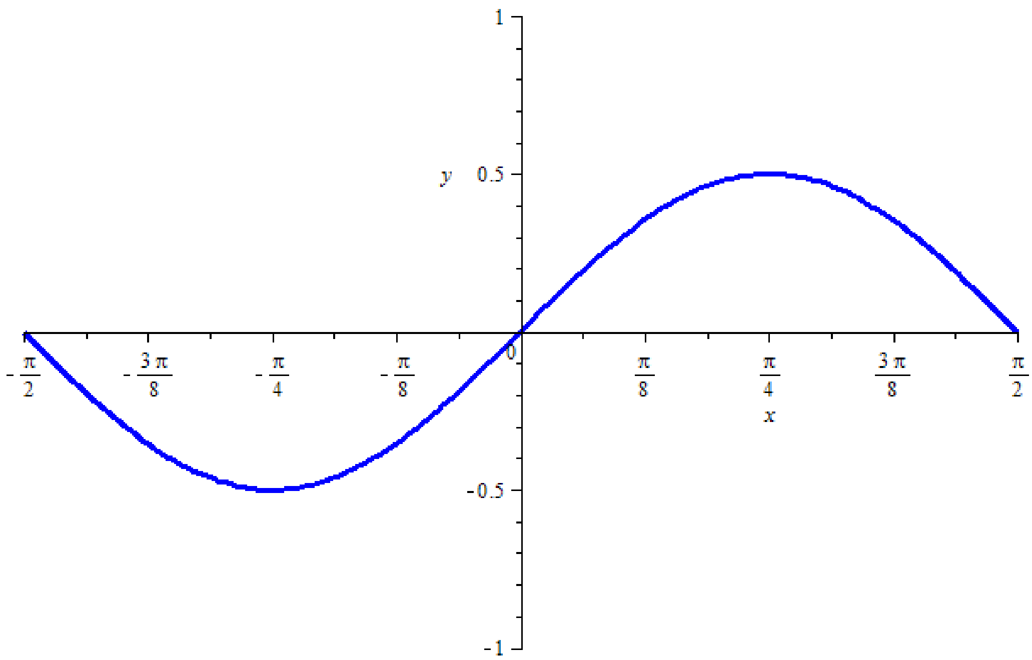
Answer on Question #67007 – Math – Calculus

Question

Is the function $sin\ x\ cos\ x$ monotonic on $[-\pi/2\ ,\pi/2]$? Justify your answer.

Solution

Using the representation $f(x) = sin\ x\ cos\ x = \frac{1}{2} sin\ 2x$ we may conclude that $f(x)$ is not monotonic on $[-\pi/2\ ,\pi/2]$. The function $f(x)$ decreases on $(-\frac{\pi}{2}, -\frac{\pi}{4})$ and on $(\frac{\pi}{4}, \frac{\pi}{2})$; the function $f(x)$ increases on $(-\frac{\pi}{4}, \frac{\pi}{4})$.



Applying Calculus the derivative of the function is

$$f'(x) = \frac{1}{2} \cdot 2 \cdot \cos 2x = \cos 2x.$$

Let us find x satisfying $f'(x) > 0$. Namely,

$$\cos 2x > 0,$$

$$-\frac{\pi}{2} + 2\pi n < 2x < \frac{\pi}{2} + 2\pi n,\ \ n \in N,$$

$$-\frac{\pi}{4} + \pi n < x < \frac{\pi}{4} + \pi n,\ \ n \in N.$$

The solution of the inequality $f'(x) > 0$ on $\left[-\frac{\pi}{2}\ ,\frac{\pi}{2}\right]$ is the interval $\left(-\frac{\pi}{4}\ ,\frac{\pi}{4}\right)$. It means the function $f(x)$ increases on $\left(-\frac{\pi}{4}\ ,\frac{\pi}{4}\right)$. The solution of the inequality $f'(x) < 0$ on $\left[-\frac{\pi}{2}\ ,\frac{\pi}{2}\right]$ consists of the intervals $\left(-\frac{\pi}{2}\ ,-\frac{\pi}{4}\right)$ and $\left(\frac{\pi}{4}\ ,\frac{\pi}{2}\right)$. Therefore, the function $f(x)$ decreases on $\left(-\frac{\pi}{2}\ ,-\frac{\pi}{4}\right)$ and on $\left(\frac{\pi}{4}\ ,\frac{\pi}{2}\right)$.

Answer: No.