Answer on Question #66421 – Math – Calculus

Question

If f(x,y) = (x^1/4+y^1/4)/(x^1/5+y^1/5), then show that

$$x \cdot \frac{\partial f}{\partial x} + y \cdot \frac{\partial f}{\partial y} = \frac{1}{20} f(x,y)$$

stating the result used.

Solution

$$f(x,y) = \frac{x^{\frac{1}{4}} + y^{\frac{1}{4}}}{x^{\frac{1}{5}} + y^{\frac{1}{5}}}$$

$$\frac{\partial f}{\partial x} = \frac{\frac{1}{4}x^{\frac{-3}{4}}\left(x^{\frac{1}{5}} + y^{\frac{1}{5}}\right) - \frac{1}{5}x^{\frac{-4}{5}}(x^{\frac{1}{4}} + y^{\frac{1}{4}})}{(x^{\frac{1}{5}} + y^{\frac{1}{5}})^2} = \frac{1}{4}\frac{x^{\frac{-3}{4}}}{(x^{\frac{1}{5}} + y^{\frac{1}{5}})} - \frac{1}{5}\frac{x^{\frac{-4}{5}}(x^{\frac{1}{4}} + y^{\frac{1}{4}})}{(x^{\frac{1}{5}} + y^{\frac{1}{5}})^2}$$

$$\frac{\partial f}{\partial y} = \frac{\frac{1}{4}y^{\frac{-3}{4}}\left(x^{\frac{1}{5}} + y^{\frac{1}{5}}\right) - \frac{1}{5}y^{\frac{-4}{5}}(x^{\frac{1}{4}} + y^{\frac{1}{4}})}{(x^{\frac{1}{5}} + y^{\frac{1}{5}})^2} = \frac{1}{4}\frac{y^{\frac{-3}{4}}}{(x^{\frac{1}{5}} + y^{\frac{1}{5}})} - \frac{1}{5}\frac{y^{\frac{-4}{5}}(x^{\frac{1}{4}} + y^{\frac{1}{4}})}{(x^{\frac{1}{5}} + y^{\frac{1}{5}})^2}$$

$$x \cdot \frac{\partial f}{\partial x} = \frac{1}{4}\frac{x^{\frac{1}{4}}}{(x^{\frac{1}{5}} + y^{\frac{1}{5}})} - \frac{1}{5}\frac{x^{\frac{1}{5}}(x^{\frac{1}{4}} + y^{\frac{1}{4}})}{(x^{\frac{1}{5}} + y^{\frac{1}{5}})^2}$$

$$y \cdot \frac{\partial f}{\partial y} = \frac{1}{4}\frac{y^{\frac{1}{4}}}{(x^{\frac{1}{5}} + y^{\frac{1}{5}})} - \frac{1}{5}\frac{y^{\frac{1}{5}}(x^{\frac{1}{4}} + y^{\frac{1}{4}})}{(x^{\frac{1}{5}} + y^{\frac{1}{5}})^2}$$

$$x \cdot \frac{\partial f}{\partial x} + y \cdot \frac{\partial f}{\partial y} = \frac{1}{4}\frac{x^{\frac{1}{4}}}{(x^{\frac{1}{5}} + y^{\frac{1}{5}})} - \frac{1}{5}\frac{x^{\frac{1}{5}}\left(x^{\frac{1}{4}} + y^{\frac{1}{4}}\right)}{(x^{\frac{1}{5}} + y^{\frac{1}{5}})^2} + \frac{1}{4}\frac{y^{\frac{1}{4}}}{(x^{\frac{1}{5}} + y^{\frac{1}{5}})} - \frac{1}{5}\frac{y^{\frac{1}{5}}\left(x^{\frac{1}{4}} + y^{\frac{1}{4}}\right)}{(x^{\frac{1}{5}} + y^{\frac{1}{5}})^2}$$

$$= \frac{1}{4}\frac{x^{\frac{1}{4}} + y^{\frac{1}{4}}}{x^{\frac{1}{5}} + y^{\frac{1}{5}}} - \frac{1}{5}\frac{x^{\frac{1}{4}} + y^{\frac{1}{4}}}{x^{\frac{1}{5}} + y^{\frac{1}{5}}} = \frac{1}{20}\frac{x^{\frac{1}{4}} + y^{\frac{1}{4}}}{x^{\frac{1}{5}} + y^{\frac{1}{5}}} = \frac{1}{20} f(x,y)$$