Answer on Question #66329 – Math – Statistics and Probability

Question

A website has on the average two hits per hour. Assuming a Poisson distribution for the number of hits per hour (X), calculate the probability that there are at most three hits.

Solution

Assume that X has a Poisson distribution with rate λ, then we have:

$$P(X = k) = \frac{\lambda^k}{k!} e^{-\lambda}, k = 0, 1, \ldots$$

(see https://en.wikipedia.org/wiki/Poisson_distribution).

Since average value of X (i.e. mathematical expectation of X) is equal to λ (see https://en.wikipedia.org/wiki/Poisson_distribution#Mean) then we have $\lambda = 2$, and

$$P(X = k) = \frac{2^k}{k!} e^{-2}, k = 0, 1, \ldots$$

Then required probability is

$$P(X \leq 3) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) = e^{-2} + 2e^{-2} + 2e^{-2} + \frac{4}{3}e^{-2} =$$

$$= \frac{19}{3e^2} \approx 0.857.$$

Answer: $\frac{19}{3e^2}$.