Answer on Question #66220 - Math - Statistics and Probability

Question

A manufacturer of automobile seats has a production line that produces an average of 100 seats per day. Because of new government regulations, a new safety device has been installed, which the manufacturer believes will reduce average daily output. A random sample of 15 days output after the installation of the safety device is shown:

93, 103, 95, 101, 91, 105, 96, 94, 101, 88, 98, 94, 101, 92, 95.

Assuming that the daily output is normally distributed, is there sufficient evidence to conclude that average daily output has decreased following the installation of the safety device?
(Use $\alpha = 0.05$)

Solution

The null hypothesis is $H_0: \mu \geq 100$.
The alternative hypothesis is $H_1: \mu < 100$
Since the claim $\mu < 100$ does not contain equality, it is an alternative hypothesis about the population mean.

Since the distribution is normal, but the population standard deviation is unknown, we will use *t-statistic* to test the null hypothesis.

The sample size is $n = 15$. The sample mean equals
$\tilde{x} = \Sigma(\text{daily output}) / \text{days} = 96.46667$.
The standard deviation of the sample equals
$s = \sqrt{(\Sigma((\text{daily output} - \tilde{x})^2)/(\text{days} - 1))} = 4.85308$.

The observed value of the statistic equals
$\tilde{t} = (96.46667 - 100) / (4.85308 / \text{sqrt}(15)) = -2.81976$.
The degree of freedoms equals
$df = 15 - 1 = 14$.
The significant level is $\alpha = 0.05$.

Method 1

The critical value for this *left-tailed test* is

$$-t_{14,0.05} = -1.761.$$

Since $\tilde{t} < -t_{14,0.05}$, we conclude that there is sufficient evidence at the 0.05 level of significance to reject the claim that the daily output has not decreased.

Method 2

The p-value for this test is

$$p = P(t < -2.81976) = 0.00682.$$

Since $p < \alpha$, we conclude that there is sufficient evidence at the 0.05 level of significance to reject the claim that the daily output has not decreased.

Answer:

Yes. There is a sufficient evidence to conclude that average daily output has decreased following the installation of the safety device.