Question

Show that
$$\lim_{x \to \infty} \left(\frac{2}{x-3}\right) = 0$$

Solution

Dividing the numerator and denominator in

$$\lim_{x\to\infty}\left(\frac{2}{x-3}\right)$$

by x we obtain

$$\lim_{x \to \infty} \left(\frac{2\frac{1}{x}}{1 - \frac{3}{x}} \right)$$

Introducing a new variable

$$u = \frac{1}{x}$$
, so $u \to 0$ as $x \to \infty$,

evaluate

$$\lim_{x \to \infty} \left(\frac{2\frac{1}{x}}{1 - \frac{3}{x}} \right) = \lim_{u \to 0} \left(\frac{2u}{-3u + 1} \right) = \frac{\lim_{u \to 0} 2u}{\lim_{u \to 0} (-3u + 1)} = \frac{2\lim_{u \to 0} u}{-3\lim_{u \to 0} u + 1} = \frac{2 \cdot 0}{-3 \cdot 0 + 1} = \frac{0}{-0 + 1}$$
$$= 0$$

We get

$$\lim_{x \to \infty} \left(\frac{2}{x-3} \right) = 0$$

Answer: 0.