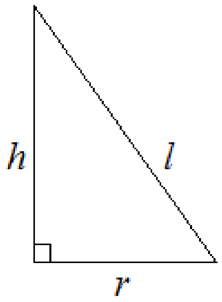
Answer on Question #66201 – Math – Calculus

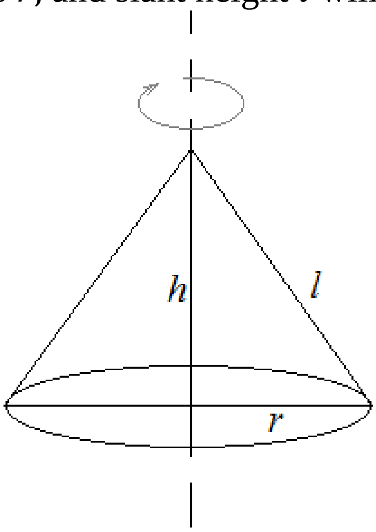
Question

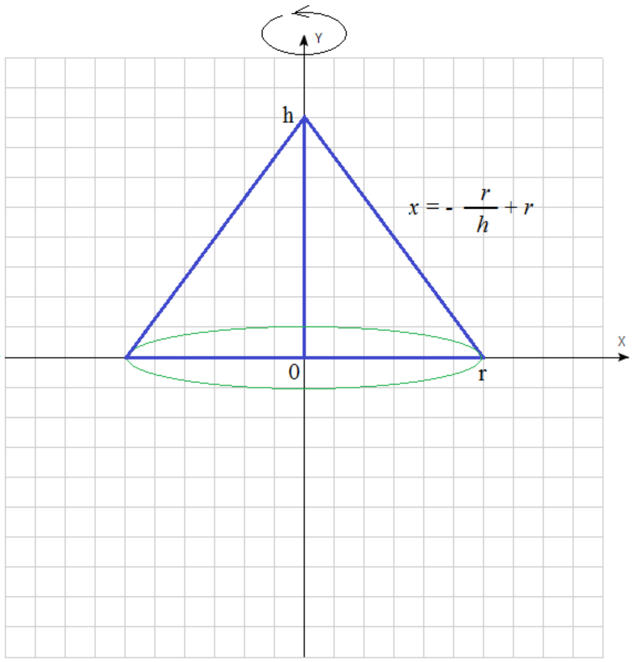
Find the volume and surface area of the solid formed by revolving a right-angled triangle about a side adjacent to the right angle. What is the solid so obtained?

Solution

When right-angled triangle is revolved about a side adjacent to the right angle, a cone with height h, radius r, and slant height l will be formed.







$x = -\frac{r}{h}y + r,\ l^2 = h^2 + r^2.$

Then the volume of the solid formed by revolving a right-angled triangle about a side adjacent to the right angle is

$$V = \pi \int_0^h x^2 dy = \pi \int_0^h (-\frac{r}{h}y + r)^2 dy = \pi \int_0^h \left(\frac{r^2}{h^2}y^2 - 2\frac{r}{h}y + r^2\right) dy =$$

$$= \pi\left(\frac{r^2}{3h^2}y^3 - \frac{r}{h}y^2 + r^2 y\right)\Big|_0^h = \pi\left(\frac{r^2}{3h^2}h^3 - \frac{r}{h}h^2 + r^2 h - 0\right) = \frac{1}{3}\pi r^2 h$$

Thus,

$$V = \frac{1}{3}\pi r^2 h.$$

The surface area of the cone equals the area of its base plus the lateral (side) surface.

$$S_{lateral} = \int_0^h 2\pi x\sqrt{1 + \left(\frac{dx}{dy}\right)^2}\,dy = \int_0^h 2\pi\left(-\frac{r}{h}y + r\right)\sqrt{1 + \left(-\frac{r}{h}\right)^2}\,dy =$$

$$= \int_0^h 2\pi\frac{\sqrt{h^2 + r^2}}{h}\left(-\frac{r}{h}y + r\right)dy = 2\pi\frac{\sqrt{h^2 + r^2}}{h}\left(-\frac{r}{2h}y^2 + ry\right)\Big|_0^h =$$

$$= 2\pi\frac{\sqrt{h^2 + r^2}}{h}\left(-\frac{r}{2h}h^2 + rh - 0\right) = \pi r\sqrt{h^2 + r^2} = \pi r l$$

$S_{base} = \pi r^2$.

The surface area of the cone equals

$S = S_{lateral} + S_{base} = \pi r^2 + \pi r l$

Thus,

$$S = \pi r^2 + \pi r l.$$

Answer: $V = \frac{1}{3}\pi r^2 h$, $S = \pi r^2 + \pi r l$.