Answer on Question #66190 – Math – Calculus

1. Which of the following statements are true or false? Give reasons for your answers.

Question

i) The function $f : R \to R$, given by $f(x) = \ln|x + \sqrt{1 + x^2}|$ is neither even nor odd.

Solution

Indeed, $f(-x) = \ln\left|-x + \sqrt{1 + (-x)^2}\right| = \ln\left|-x + \sqrt{1 + x^2}\right|$ and neither $f(-x) = f(x)$ nor $f(-x) = -f(x)$, so f is neither even nor odd.

Answer: True.

Question

ii) $\frac{d}{dx}\int_x^0 \sin(t^2)dt = -\sin x^2$.

Solution

Indeed, if $F(t)$ is some primitive for $f(t) = \sin t^2$, then by formula of Newton-Leibniz $\int_x^0 \sin(t^2)dt = F(0) - F(x)$.

Therefore,

$$\frac{d}{dx}\int_x^0 \sin(t^2)dt = \frac{d}{dx}(F(0) - F(x)) = -F'(x) = -\sin x^2.$$

Answer: True.

Question

iii) The area enclosed by the x-axis and the curve $y = cosx$ over the interval $\left[-\frac{\pi}{2}, \frac{3\pi}{2}\right]$ is 0.

Solution

The area enclosed by the x-axis and the curve $y = cosx$ over the interval $\left[-\frac{\pi}{2}, \frac{3\pi}{2}\right]$ is calculated by formula

$$A = \int_{-\pi/2}^{\pi/2} (\cos x - 0)\, dx + \int_{\pi/2}^{3\pi/2} (0 - \cos x)\, dx = \sin x|_{-\pi/2}^{\pi/2} - \sin x|_{\pi/2}^{3\pi/2}$$
$$= \big(1 - (-1)\big) - (-1 - 1) = 2 + 2 = 4.$$

Answer: False

Question

iv) If f and g are functions over R such that $f + g$ is continuous, then f must be continuous.

Solution

The counterexample can be the following:

$f(x) = \begin{cases} 0, & x < 0, \\ 1, & x \geq 0; \end{cases}$ and $g(x) = \begin{cases} 1, & x < 0, \\ 0, & x \geq 0. \end{cases}$

Then $f(x) + g(x) = 1$ for all $x \in R$ and therefore $f + g$ is continuous on R but f is not continuous at the point 0.

Answer: False

Question

v) $x - y + 2 = 0$ is a tangent to the curve $(x + y)^3 = (x - y + 2)^2$ at (−1, 1).

Solution

Let us consider the function

$$F(x,y) = (x + y)^3 - (x - y + 2)^2.$$

We search for the derivative of the implicit function $y = y(x)$ at the point $(-1,1)$ given by the equation

$$F(x,y) = 0.$$

We have

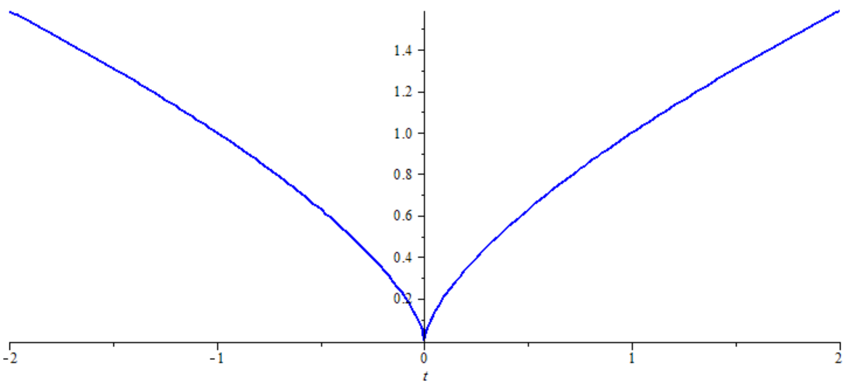
$$F'_y(x,y) = 3(x+y)^2 + 2(x - y + 2).$$

Then

$$F'_y(-1,1) = 0,$$

and therefore y'(-1) is undefined. Hence the line $x - y + 2 = 0$ cannot be a tangent to the curve. We also can show that $(-1,1)$ is a turning point of the curve, hence $x - y + 2 = 0$ is not a tangent to the curve $(x + y)^3 = (x - y + 2)^2$ at (−1, 1).

Let us make the linear change of variables z$= x + y$, $t = x - y + 2$, which the point $(-1,1)$ moves to the origin $(0,0)$. Then in new variables the curve is given by the equation $z^3 = t^2$. This equation describes the semicubical parabola, which has a turning point at the origin.



Answer: False.

Answer provided by https://www.AssignmentExpert.com