Answer on Question #66177 – Math – Statistics and Probability

Question

Consider a small post office with a single staff member operating a single postal counter.

Suppose that the probability p_k, that there are k customers in the post office, is given by

$$p_k = p_0 p^k, k = 0,1,2,\ldots,$$

where $0 < p < 1$.

(a) Show that

$$p_0 = 1 - p.$$

(b) Determine the probability that a newly arriving customer has to wait to be served.

Solution

(a) Probabilities of all possible outcomes must sum up to 1, hence $p_0 + p_1 + \ldots + p_n + \ldots = 1$.

Substituting formula for $p_k = p_0 p^k, k = 0,1,2,\ldots$, we can rewrite the previous sum

$p_0 + p_0 p + \ldots + p_0 p^n + \ldots = p_0(1 + p + \ldots + p^n + \ldots)$.

If $0 < p < 1$, then

$$1 + p + \ldots + p^n + \ldots = \frac{1}{1-p}$$

by the formula for the sum of the infinite arithmetic progression.

It follows from the initial equation

$$p_0 + p_1 + \ldots + p_n + \ldots = \frac{p_0}{1-p} = 1$$

that

$$p_0 = 1 - p.$$

(b) Customer has to wait in case when there are some other customers in the office. The opposite to this event is 'no customer at the moment'. So

$$P(\text{have to wait}) = 1 - P(k = 0) = 1 - p_0 = 1 - 1 + p = p.$$

Answer:

(a) $p_0 = 1 - p$;

(b) $P(\text{have to wait}) = p$.