ANSWER ON QUESTION #66145 – Math – Differential Equations

QUESTION

Given that $y_1 = \frac{1}{x}$ is one solution of the differential equation

$$2x^2y" + 3xy' - y = 0, x > 0,$$

find a second linearly independent solution of the equation.

SOLUTION

We transform the original equation to the form

$$y'' + p(x)y' + q(x)y = 0$$

$$2x^2y" + 3xy' - y = 0 \Big| : \frac{1}{2x^2} \to y'' + \frac{3x}{2x^2}y' - \frac{y}{2x^2} = 0$$

$$y'' + \underbrace{\frac{3}{2x}}_{p(x)} y' \underbrace{- \frac{1}{2x^2}}_{q(x)} y = 0 \qquad (1)$$

$y_1(x) = \frac{1}{x}$ is the first solution of the differential equation (1)

Using the Liouville formula (https://en.wikipedia.org/wiki/Liouville%27s_formula)
we can find a second solution:

$$y_2(x) = y_1 \int \frac{e^{-\int p(x)dx}}{y_1^2} dx = \frac{1}{x} \int \frac{e^{-\int \frac{3}{2x}dx}}{\frac{1}{x^2}} dx = \frac{1}{x} \int x^2 e^{-\frac{3}{2}\ln(x)} dx$$

$$= \frac{1}{x} \int x^2 e^{\ln\left(x^{-\frac{3}{2}}\right)} dx = \frac{1}{x} \int x^2 x^{-\frac{3}{2}} dx = \frac{1}{x} \int x^{2-\frac{3}{2}} dx = \frac{1}{x} \int x^{\frac{1}{2}} dx$$

$$= \frac{1}{x} \cdot \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} = \frac{1}{x} \cdot \frac{x^{\frac{3}{2}}}{\frac{3}{2}} = \frac{2}{3} \cdot \frac{x^{\frac{3}{2}}}{x} = \frac{2}{3} \cdot x^{\frac{1}{2}} \equiv \frac{2\sqrt{x}}{3}.$$

ANSWER:

$y_2(x) = \frac{2\sqrt{x}}{3}$ is a second linearly independent solution of the equation

Answer provided by https://www.AssignmentExpert.com