Answer on Question #66142 – Math – Differential Equations

Question

Write the ordinary differential equation ydx+(xy+x-3y)dy=0 in the linear form, and hence find its solution

Solution

We have the differential equation

$$ydx + (xy + x - 3y)dy = 0 \qquad (1)$$

Dividing equation (1) by ydy we get

$$\frac{dx}{dy} + \left(1 + \frac{1}{y}\right)x = 3 \qquad (2)$$

This equation (2) has the linear form:

$$\frac{dx}{dy} + P(y)x = Q(y) \qquad (3)$$

We need to multiply both sides by the integrating factor

$$I(y) = \exp\left(\int P(y)dy\right)$$

and integrate both sides. For this problem we have

$$I(y) = \exp\left(\int \left(1 + \frac{1}{y}\right) dy\right) = \exp(y + \ln y) = ye^y$$

Multiplying both sides of the differential equation (2) by ye^y, we get

$$ye^y \frac{dx}{dy} + ye^y \left(1 + \frac{1}{y}\right) x = 3ye^y$$

or

$$ye^y \frac{dx}{dy} + e^y(y + 1)x = 3ye^y$$

or

$$\frac{d}{dy}(xye^y) = 3ye^y$$

Integrating both sides, we get

$$xye^y = 3\int ye^y\, dy = 3\left(ye^y - \int e^y\, dy\right) = 3(ye^y - e^y) + C$$

Dividing equation by ye^y, we get a solution of the initial equation

$$x = 3\left(1 - \frac{1}{y}\right) + \frac{C}{y}e^{-y}$$

Answer: $\frac{dx}{dy} + \left(1 + \frac{1}{y}\right)x = 3$; $x = 3\left(1 - \frac{1}{y}\right) + \frac{C}{y}e^{-y}$.