Answer on Question #66108 - Math – Differential Equations

Question

The pde uxx+ x^2 uxy -(x^2/2+1/4) uyy = 0 is hyperbolic in the entire xy-plane. true or false, why?

Solution

We have

$$u_{xx} + x^2 u_{xy} - \left(\frac{x^2}{2} + \frac{1}{4}\right) u_{yy} = 0 \quad (1)$$

which is a particular case of the linear second-order partial differential equation

$$Au_{xx} + Bu_{xy} + Cu_{yy} + Du_x + Eu_y + F = 0$$

where coefficients A, B, C, D, E and free term F in general are functions of the independent variables x, y , but do not depend on the unknown function u.

This equation is said to be hyperbolic if [1, page 435]

$$B^2 - 4AC > 0 \quad (2)$$

For the given equation (1)

$$A = 1, \ \ B = x^2, \ \ C = -\left(\frac{x^2}{2} + \frac{1}{4}\right), \ \ D = E = F = 0$$

Substituting values A, B, C in (2) we get

$$B^2 - 4AC = x^4 + 4\left(\frac{x^2}{2} + \frac{1}{4}\right) = x^4 + 2x^2 + 1 = (x^2 + 1)^2 > 0$$

This inequality holds for any x. Thus, this equation is hyperbolic in the entire *xy* -plane.

Answer: True. The pde $u_{xx} + x^2 u_{xy} - \left(\frac{x^2}{2} + \frac{1}{4}\right) u_{yy} = 0$ is hyperbolic in the entire xy-plane.

References:

[1] Dennis G. Zill, Michael R. Cullen. Differential equations, seventh edition.