Answer on Question 66106 - Math - Calculus

Question: The normal form of the differential equation

$$y'' - 4xy' + (4x^2 - 1)y = -3e^{x^2} \sin 2x \quad \text{is}$$

$$\frac{d^2v}{dx^2} + v = -3 \sin 2x, \quad \text{where} \quad v = ye^{-x^2}.$$

True or false why?

Answer: True.

Solution: As $v = ye^{-x^2}$, then $y = ve^{x^2}$ and

$$y' = (v' + 2xv)e^{x^2}$$
$$y'' = (v'' + 2xv' + 2v + 2x(v' + 2xv))e^{x^2} = (v'' + 4xv' + (4x^2 + 2)v)e^{x^2}.$$

Substituting y, y' and y'' into differential equation yields

$$(v'' + 4xv' + (4x^2 + 2)v)e^{x^2} - 4x(v' + 2xv)e^{x^2} + (4x^2 - 1)ve^{x^2} = -3e^{x^2} \sin 2x$$

and

$$v'' + 4xv' + (4x^2 + 2)v - 4xv' - 8x^2v + (4x^2 - 1)v = -3 \sin 2x$$
$$v'' + (4x^2 + 2 - 8x^2 + 4x^2 - 1)v = -3 \sin 2x$$
$$v'' + v = -3 \sin 2x.$$