Answer on Question #66105 – Math – Calculus

Question

The initial value problem

$$\frac{dy}{dx} = x^2 + y^2 \ ; \ \ y(0) = 0$$

has a unique solution in some interval of the form $-h < x < h$.
True or false, why?

Solution

<u>Existence and Uniqueness Theorem for First Order ODE's</u> [1, page 150]:
Let R be a rectangle and let $f(x, y)$ be continuous throughout R and satisfy the Lipschitz Condition with respect to y throughout R. Let (x_0, y_0) be an interior point of R. Then there exists an interval containing x_0 on which there exists a unique function $y(x)$ satisfying $y' = f(x, y)$ and $y(x_0) = y_0$.

We have
$f(x, y) = x^2 + y^2$; $\frac{df}{dy} = 2y$,
hence $f(x, y)$ has a continuous derivative with respect to y, therefore $f(x, y)$ satisfies the Lipschitz Condition with respect to y throughout rectangle R [2, Proposition 1].

Since $f(x, y)$ satisfies the conditions of Existence and Uniqueness Theorem, the initial value problem

$$\frac{dy}{dx} = x^2 + y^2 \ ; \ \ y(0) = 0$$

has a unique solution in some interval of the form $-h < x < h$.

Answer: true.

References:
[1] Differential Equations I, MATB44H3F, Version September 15, 2011-1949.
[2] Lipschitz condition and differentiability. Retrieved from
http://planetmath.org/lipschitzconditionanddifferentiability