

Answer on Question #66104 – Math – Calculus

Question

The orthogonal trajectories of all the parabolas with vertices at the origin and foci on the x-axis

$$x^2 + 2y^2 = c^2$$

True or false. Why?

Solution

The equation of the parabola:

$$x = ay^2, \text{ } a \text{ is a real constant .}$$

Then

$$1 = ayy'$$

$$y' = \frac{1}{ay}$$

$$a = \frac{x}{y^2}$$

$$y' = \frac{y^2}{xy} = \frac{y}{x}$$

Changing y' into $-1/y'$ one gets

$$-\frac{1}{y'} = \frac{y}{x}$$

$$y' = -\frac{x}{y}$$

$$ydy = -x dx$$

$$\int ydy = - \int x dx$$

The equation of the orthogonal trajectories is

$$\frac{x^2}{2} + \frac{y^2}{2} = C$$

or

$$x^2 + y^2 = c^2$$

Answer: False