Answer on Question #66098 – Math – Calculus

Question

Evaluate

$$\iiint_S z^2\,dxdydz\,,$$

where S is the solid region between the spheres ρ =1 and ρ = 2 by using spherical coordinates.

Solution

Using spherical coordinates ρ, θ, φ, where

$$x = \rho\sin\theta\cos\varphi\,,$$

$$y = \rho\sin\theta\sin\varphi\,,$$

$$z = \rho\cos\theta\,,$$

$$dxdydz = \rho^2\sin\theta d\theta d\varphi d\rho\,,$$

evaluate

$$\iiint_S z^2\,dxdydz = \int_1^2\int_0^{2\pi}\int_0^{\pi}\rho^2\cos^2\theta\cdot\rho^2\sin\theta d\theta d\varphi d\rho = \int_1^2\int_0^{2\pi}\int_0^{\pi}\rho^4\cos^2\theta\sin\theta d\theta d\varphi d\rho =$$

$$= -\int_1^2\int_0^{2\pi}\int_0^{\pi}\rho^4\cos^2\theta d(\cos\theta)d\varphi d\rho = -\int_1^2\rho^4 d\rho\int_0^{2\pi}d\varphi\int_0^{\pi}\cos^2\theta d(\cos\theta) = -\left.\frac{\rho^5}{5}\right|_1^2\cdot\left.\varphi\right|_0^{2\pi}\cdot\left.\frac{\cos^3\theta}{3}\right|_0^{\pi} =$$

$$= -\frac{2^5-1^5}{5}\cdot(2\pi-0)\cdot\frac{\cos^3\pi-\cos^3 0}{3} = -\frac{32-1}{5}\cdot 2\pi\cdot\frac{(-1)^3-1^3}{3} = -\frac{31}{5}\cdot 2\pi\cdot\frac{-2}{3} = \frac{124\pi}{15}\,.$$

Answer: $\frac{124\pi}{5}$.