Answer on Question #66097 – Math – Calculus

Question

Evaluate

$$\iint_D (x+2y)\,dA,$$

where D is the region bounded by the parabolas $y = 2x^2$ and $y = 1 + x^2$

Solution

If $f(x,y)$ is continuous on a type I region D such that

$$D = \{(x,y) \mid a \le x \le b, \quad g_1(x) \le y \le g_2(x)\}$$

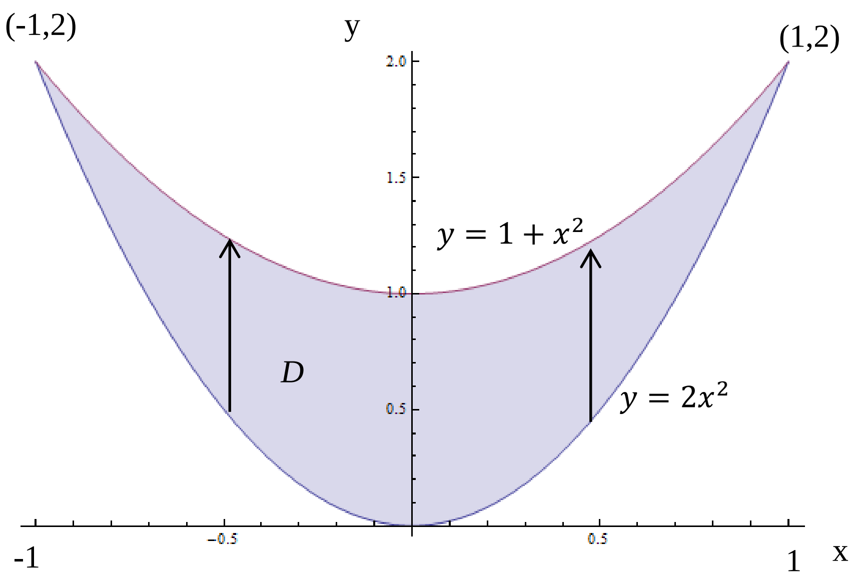
Then

$$\iint_D f(x,y)\,dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)\,dy\,dx \text{ [1, page 1002].}$$

Functions $g_1(x) = 2x^2$ and $g_2(x) = 1 + x^2$ are given. We need to find a and b. The given parabolas intersect when $2x^2 = 1 + x^2$, hence $x^2 = 1$, so $x = \pm 1$ and $y = 2 \cdot (\pm 1)^2 = 2$

Thus, the region D is $D = \{(x,y) | -1 \le x \le 1,\ 2x^2 \le y \le 1 + x^2\}$.

The region D is given below.



$$\iint_D (x+2y)\,dA = \int_{-1}^{1} \int_{2x^2}^{1+x^2} (x+2y)\,dy\,dx = \int_{-1}^{1} (xy + y^2)\Big|_{2x^2}^{1+x^2} dx$$

$$= \int_{-1}^{1} \left((x(1+x^2) + (1+x^2)^2) - (x(2x^2) + (2x^2)^2) \right) dx$$

$$= \int_{-1}^{1} \left((x + x^3 + 1 + 2x^2 + x^4) - (2x^3 + 4x^4) \right) dx$$

$$= \int_{-1}^{1} (1 + x + 2x^2 - x^3 - 3x^4) dx = \left(x + \frac{x^2}{2} + \frac{2x^3}{3} - \frac{x^4}{4} - \frac{3x^5}{5} \right) \Bigg|_{-1}^{1}$$

$$= \left(1 + \frac{1}{2} + \frac{2}{3} - \frac{1}{4} - \frac{3}{5} \right) - \left(-1 + \frac{1}{2} - \frac{2}{3} - \frac{1}{4} + \frac{3}{5} \right)$$

$$= 1 + \frac{1}{2} + \frac{2}{3} - \frac{1}{4} - \frac{3}{5} + 1 - \frac{1}{2} + \frac{2}{3} + \frac{1}{4} - \frac{3}{5} = 2 + \frac{4}{3} - \frac{6}{5} = \frac{32}{15} \approx 2.13333$$

Answer: $\iint_D (x + 2y) dA = \frac{32}{15}$.

References:

[1] James Stewart. Calculus: Early Transcendentals, Eighth Edition [2015].