Answer on Question #66096 – Math – Calculus

Question

Check if the following integrals are independent of path and evaluate those which are independent:

$$\boldsymbol{i})\ \iint\limits_{(0,0)}^{(3,4)} (6xy - y^3)dx + (3x^2 - x^3y)dy$$

$$\boldsymbol{ii})\ \iint\limits_{(-1,4)}^{(3,8)} (3x^2 - 2y^2)dx + (-4xy)dy$$

Solution

$\boldsymbol{i})$ *Check the independence of path*:

$$P(x, y) = 6xy - y^3$$

$$Q(x, y) = 3x^2 - x^3y$$

$$\frac{\partial P}{\partial y} = 6x - 3y^2$$

$$\frac{\partial Q}{\partial x} = 6x - 3x^2y$$

$$\frac{\partial P}{\partial y} \neq \frac{\partial Q}{\partial x}, hence\ the\ integral\ depends\ \ on\ the\ path\ of\ integration$$

$\boldsymbol{ii})$ *Check the independence of path*:

$$P(x, y) = 3x^2 - 2y^2$$

$$Q(x, y) = -4xy$$

$$\frac{\partial P}{\partial y} = -4y$$

$$\frac{\partial Q}{\partial x} = -4y$$

$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}, hence\ the\ integral\ is\ independent\ \ of\ the\ path\ of\ integration$$

$(3x^2 - 2y^2)dx + (-4xy)dy = d(x^3) - d(2y^2x) = d(x^3 - 2y^2x) = du$

$So, the\ field\ potential\ is\ u = x^3 - 2y^2x$

$Then\ by\ formula$

$$\int_{AB} Pdx + Qdy = u(B) - u(A)$$

$compute$

$$\iint_{(-1,4)}^{(3,8)} (3x^2 - 2y^2)dx + (-4xy)dy = u(3,8) - u(-1,4)$$

$$= (27 - 2 * 64 * 3) - \big(-1 - 2 * 16 * (-1)\big) = -357 + 31 = -326$$

Answer: **(i)** *the integral is dependent of the path*; **(ii)** *the integral is independent of the path and*

$\iint_{(-1,4)}^{(3,8)}(3x^2 - 2y^2)dx + (-4xy)dy = -326.$