Answer on Question #66072 – Math – Calculus

Question

Find the centre of gravity of a mass in the shape of a semicircular disc of radius 4, if the density at (x, y) is

$$\frac{2y}{x^2 + y^2}$$

Solution

The coordinates of the centre of gravity of a mass:

$$x_c = \frac{\iint \gamma(x,y)xdxdy}{\iint \gamma(x,y)dxdy}, \qquad y_c = \frac{\iint \gamma(x,y)ydxdy}{\iint \gamma(x,y)dxdy},$$

where $\gamma(x, y)$ is density.

Using polar coordinates (r, φ), substitute $x = r\cos\varphi$, $y = r\sin\varphi$:

$\gamma(x, y) = \frac{2y}{x^2+y^2} = \frac{2r\sin\varphi}{r^2} = \frac{2\sin\varphi}{r}$,

$\iint \gamma(x, y)dxdy = 2\int_0^\pi \sin\varphi\, d\varphi \int_0^4 dr = -8\cos\varphi|_0^\pi = 16$,

$\iint \gamma(x, y)xdxdy = \int_0^\pi 2\sin\varphi\cos\varphi\, d\varphi \int_0^4 rdr = -8 \cdot \frac{\cos 2\varphi}{2}\Big|_0^\pi = 0$,

$\iint \gamma(x, y)ydxdy = 2\int_0^\pi (\sin\varphi)^2 d\varphi \int_0^4 rdr = 16\int_0^\pi \frac{1-\cos 2\varphi}{2} d\varphi = 8\left(\varphi - \frac{\sin 2\varphi}{2}\right)\Big|_0^\pi = 8\pi$.

Then the coordinates of the centre of gravity are

$x_c = \frac{0}{16} = 0$,

$y_c = \frac{8\pi}{16} = \frac{\pi}{2}$.

Answer: $x_c = 0,\ y_c = \frac{\pi}{2}$.

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