Answer on Question #66071 – Math – Calculus

Question

Write $\int_0^1 \int_0^{\sqrt{1-x^2}} \sqrt{1-y^2}\,dydx$ as an integral over a region D. Sketch the region D and show that it is of both types 1 and 2. Reverse the order of integration and evaluate it.

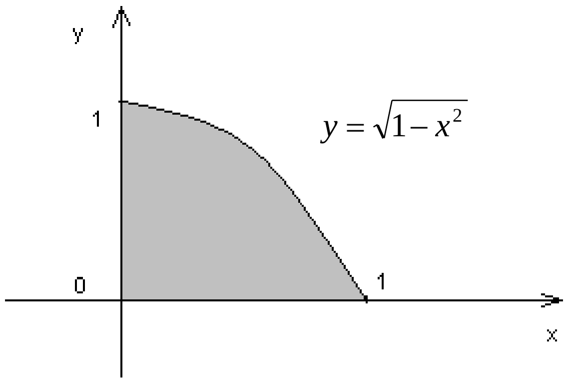
Solution

The integral $\int_0^1 \int_0^{\sqrt{1-x^2}} \sqrt{1-y^2}\,dydx$ is given.

First write this integral in the form of double integral:

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \sqrt{1-y^2}\,dydx = \iint_D \sqrt{1-y^2}\,dydx,$$

where D is the region that is depicted in figure below:



Region D can be defined in two ways:

1) $0 \le x \le 1,\ 0 \le y \le \sqrt{1-x^2}$;
2) $0 \le y \le 1,\ 0 \le x \le \sqrt{1-y^2}$.

Then reverse the order of integration and evaluate the initial integral:

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \sqrt{1-y^2}\,dydx = \iint_D \sqrt{1-y^2}\,dydx = \int_0^1 \int_0^{\sqrt{1-y^2}} \sqrt{1-y^2}\,dxdy = \int_0^1 \sqrt{1-y^2}\,x\Big|_0^{\sqrt{1-y^2}} dy = \int_0^1 \sqrt{1-y^2}\sqrt{1-y^2}\,dy =$$

$$= \int_0^1 (1-y^2)dy = (y-\frac{y^3}{3})\Big|_0^1 = 1-\frac{1}{3} = \frac{2}{3}.$$

Answer: $\int_0^1 \int_0^{\sqrt{1-x^2}} \sqrt{1-y^2}\,dydx = \iint_D \sqrt{1-y^2}\,dydx = \int_0^1 \int_0^{\sqrt{1-y^2}} \sqrt{1-y^2}\,dxdy = \frac{2}{3}$.

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