Answer on Question #66044 – Math – Calculus

Question

Find the values of a and b for which the following limit exists: $\lim\limits_{x\to 0}\frac{ae^x+e^{-x}+bx}{1-\cos x}$.

Solution

Obviously the limit of the denominator is

$$(1-\cos x)|_{x=0}=1-\cos(0)=0.$$

Then the limit of the numerator also must be equal to zero.

On the other hand, the limit of numerator is

$$(ae^x+e^{-x}+bx)|_{x=0}=ae^0+e^{-0}+b\cdot 0=a+1.$$

Then we have

$$a+1=0\Leftrightarrow a=-1.$$

We have the following limit:

$$\lim_{x\to 0}\frac{-e^x+e^{-x}+bx}{1-\cos x}.$$

Let us compute it using L'Hôpital's rule (see https://en.wikipedia.org/wiki/L'Hôpital's_rule):

$$\lim_{x\to 0}\frac{-e^x+e^{-x}+bx}{1-\cos x}=\lim_{x\to 0}\frac{(-e^x+e^{-x}+bx)'}{(1-\cos x)'}=\lim_{x\to 0}\frac{-e^x-e^{-x}+b}{\sin x}.$$

The limit of denominator is equal to 0 again.

Then we obtain the following condition:

$$(-e^x-e^{-x}+b)|_{x=0}=-e^0-e^{-0}+b=0\Leftrightarrow -2+b=0\Leftrightarrow b=2.$$

So we get

$$\begin{cases} a=-1, \\ b=2. \end{cases}$$

Let us check that the original limit exists using L'Hôpital's rule:

$$\lim_{x\to 0}\frac{-e^x+e^{-x}+2x}{1-\cos x}=\lim_{x\to 0}\frac{(-e^x+e^{-x}+2x)'}{(1-\cos x)'}=\lim_{x\to 0}\frac{-e^x-e^{-x}+2}{\sin x}=\lim_{x\to 0}\frac{-e^x+e^{-x}}{\cos x}=\frac{-1+1}{1}=0<\infty.$$

Answer: $\begin{cases} a=-1 \\ b=2 \end{cases}$.