Answer on Question #66043 – Math – Calculus

Question

Apply Inverse function theorem to check the local invertibility of the following function $f: R^2 \to R^2$ given by

$$F(x,y) = \begin{pmatrix} y\cos x \\ x - y + 2 \end{pmatrix}$$

at the point $(0, \pi)$.

Solution

Jacobian matrix:

$$J_F(x,y) = \begin{pmatrix} \frac{\partial(y\cos x)}{\partial x} & \frac{\partial(y\cos x)}{\partial y} \\ \frac{\partial(x-y+2)}{\partial x} & \frac{\partial(x-y+2)}{\partial y} \end{pmatrix} = \begin{pmatrix} -y\sin x & \cos x \\ 1 & -1 \end{pmatrix};$$

$$det(J_F) = \begin{vmatrix} -y\sin x & \cos x \\ 1 & -1 \end{vmatrix} = y\sin x - \cos x.$$

At the point $(0, \pi)$:

$$det(J_F) = \pi\sin 0 - \cos 0 = -1 \neq 0,$$

hence $F(x,y)$ is invertible at the point $(0, \pi)$.

Answer: $F(x,y)$ is invertible at the point $(0, \pi)$.