Answer on Question #66040 – Math – Calculus

Question

Evaluate: $\lim_{x\to\infty}\left(\sqrt{2x^2+3x-2}-\sqrt{2x^2-3x+2}\right)$

Solution

$$\lim_{x\to\infty}\left(\sqrt{2x^2+3x-2}-\sqrt{2x^2-3x+2}\right)=$$

$$=\lim_{x\to\infty}\frac{\left(\sqrt{2x^2+3x-2}-\sqrt{2x^2-3x+2}\right)\left(\sqrt{2x^2+3x-2}+\sqrt{2x^2-3x+2}\right)}{\left(\sqrt{2x^2+3x-2}+\sqrt{2x^2-3x+2}\right)}=$$

$$=\lim_{x\to\infty}\frac{\left(\sqrt{2x^2+3x-2}\right)^2-\left(\sqrt{2x^2-3x+2}\right)^2}{\left(\sqrt{2x^2+3x-2}+\sqrt{2x^2-3x+2}\right)}=\lim_{x\to\infty}\frac{2x^2+3x-2-\left(2x^2-3x+2\right)}{\left(\sqrt{2x^2+3x-2}+\sqrt{2x^2-3x+2}\right)}=$$

$$=\lim_{x\to\infty}\frac{2x^2+3x-2-2x^2+3x-2}{\left(\sqrt{2x^2+3x-2}+\sqrt{2x^2-3x+2}\right)}=\lim_{x\to\infty}\frac{6x-4}{\left(\sqrt{2x^2+3x-2}+\sqrt{2x^2-3x+2}\right)}=$$

$$=\lim_{x\to\infty}\frac{x\left(6-\frac{4}{x}\right)}{x\left(\sqrt{2+\frac{3}{x}-\frac{2}{x^2}}+\sqrt{2-\frac{3}{x}+\frac{2}{x^2}}\right)}=\lim_{x\to\infty}\frac{\left(6-\frac{4}{x}\right)}{\left(\sqrt{2+\frac{3}{x}-\frac{2}{x^2}}+\sqrt{2-\frac{3}{x}+\frac{2}{x^2}}\right)}=$$

$$=\begin{bmatrix}\text{note that}\\ \frac{4}{x}\to 0 \text{ as } x\to\infty\\ \frac{3}{x}\to 0 \text{ as } x\to\infty\\ \frac{2}{x^2}\to 0 \text{ as } x\to\infty\end{bmatrix}=\frac{6}{\sqrt{2}+\sqrt{2}}=\frac{6}{2\sqrt{2}}=\frac{3}{\sqrt{2}}=\frac{3\sqrt{2}}{2}.$$

Answer: $\frac{3\sqrt{2}}{2}$.