Answer on Question #66012 – Math – Calculus

Question

We find the directional derivative of

$$f(x,y)=\begin{cases}\dfrac{2xy}{x^2+y^2},(x,y)\neq(0,0)\\ 0,(x,y)=(0,0)\end{cases}$$

at (0,0) in the direction of

(i) $\theta=\dfrac{\pi}{2}$,

(ii) $\theta=\dfrac{\pi}{4}$.

Solution

We shall use the formula

$$\frac{\partial f}{\partial l}\left(\vec{a}\right)=\lim_{h\to 0}\frac{f\left(\vec{a}+h\cdot\vec{l}\right)-f\left(\vec{a}\right)}{h}.$$

(i)

$$\vec{l}=\left(\cos\theta,\cos\left(\frac{\pi}{2}-\theta\right)\right)=\left(\cos\frac{\pi}{2},\cos 0\right)=(0,1).$$

$$\frac{\partial f}{\partial l}\left(\vec{0}\right)=\lim_{h\to 0}\frac{f\left(\vec{0}+h\cdot\vec{l}\right)-f\left(\vec{0}\right)}{h}=\lim_{h\to 0}\frac{f\left(h\cdot\vec{l}\right)-0}{h}=\lim_{h\to 0}\frac{f(0,h)}{h}=\lim_{h\to 0}\frac{\frac{2\cdot 0\cdot h}{0^2+h^2}}{h}=\lim_{h\to 0}\frac{0}{h}=0.$$

Hence, $\dfrac{\partial f}{\partial l}(0,0)=0$.

(ii)

$$\vec{l}=\left(\cos\theta,\cos\left(\frac{\pi}{2}-\theta\right)\right)=\left(\cos\frac{\pi}{4},\cos\frac{\pi}{4}\right)=\left(\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right),$$

$$\frac{\partial f}{\partial l}\left(\vec{0}\right)=\lim_{h\to 0}\frac{f\left(\vec{0}+h\cdot\vec{l}\right)-f\left(\vec{0}\right)}{h}=\lim_{h\to 0}\frac{f\left(h\cdot\vec{l}\right)-0}{h}=\lim_{h\to 0}\frac{f\left(\frac{\sqrt{2}}{2}h,\frac{\sqrt{2}}{2}h\right)}{h}=$$

$$=\lim_{h\to 0}\frac{\frac{2\cdot\frac{\sqrt{2}}{2}h\cdot\frac{\sqrt{2}}{2}h}{\left(\frac{\sqrt{2}}{2}h\right)^2+\left(\frac{\sqrt{2}}{2}h\right)^2}}{h}=\lim_{h\to 0}\frac{\frac{h^2}{2\left(\frac{\sqrt{2}}{2}h\right)^2}}{h}=\lim_{h\to 0}\frac{\frac{h^2}{2\cdot\frac{2}{4}h^2}}{h}=\lim_{h\to 0}\frac{1}{h}=\infty.$$

Hence, $\dfrac{\partial f}{\partial l}(0,0)$ does not exist.

Answer: (i) 0; **(ii)** does not exist.

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