Answer on Question #66011 – Math – Calculus

Question

If

$$z = e^x \sin y,$$

where

$$x = st^2 \text{ and } y = ts^2,$$

find

$$\frac{\partial z}{\partial s}, \frac{\partial z}{\partial t}$$

Solution

Using

$\frac{\partial x}{\partial t} = \frac{\partial}{\partial t}(st^2) = 2ts,\ \frac{\partial x}{\partial s} = \frac{\partial}{\partial s}(st^2) = t^2,$

$\frac{\partial y}{\partial t} = \frac{\partial}{\partial t}(ts^2) = s^2,\quad \frac{\partial y}{\partial s} = \frac{\partial}{\partial s}(ts^2) = 2ts,$

$\frac{\partial z}{\partial x} = \frac{\partial}{\partial x}(e^x \sin y) = e^x \sin y,\ \frac{\partial z}{\partial y} = \frac{\partial}{\partial y}(e^x \sin y) = e^x \cos y,$

compute

$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial s} = e^x \cdot \sin y \cdot \frac{\partial x}{\partial s} + e^x \cdot \cos y \cdot \frac{\partial y}{\partial s} = e^x \cdot \sin y \cdot t^2 + e^x \cdot \cos y \cdot 2ts,$

$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial t} = e^x \cdot \sin y \cdot \frac{\partial x}{\partial t} + e^x \cdot \cos y \frac{\partial y}{\partial t} = e^x \cdot \sin y \cdot 2ts + e^x \cdot \cos y \cdot s^2.$

Substituting $x = st^2$ and $y = ts^2$ express the answer explicitly through the parameters s and t :

$\frac{\partial z}{\partial s} = e^{st^2} \cdot t^2 \cdot \sin(ts^2) + 2ts \cdot e^{st^2} \cdot \cos(ts^2),$

$\frac{\partial z}{\partial t} = e^{st^2} \cdot 2ts \cdot \sin(ts^2) + e^{st^2} \cdot s^2 \cdot \cos(ts^2)\,.$

Answer: $\frac{\partial z}{\partial s} = e^{st^2} \cdot t^2 \cdot \sin(ts^2) + 2ts \cdot e^{st^2} \cdot \cos(ts^2),$

$\frac{\partial z}{\partial t} = e^{st^2} \cdot 2ts \cdot \sin(ts^2) + e^{st^2} \cdot s^2 \cdot \cos(ts^2)\,.$