Question

Prove that the functions g(x, y)=(2x-3y)/(4x+5y) and h(x,y)=x/y, $y\neq 0$, $y\neq -(4/5)x$ are functionally dependent.

Solution

 $g(x, y) = \frac{2x - 3y}{4x + 5y}$ and $h(x, y) = \frac{x}{y}$.

By definition, two functions are called functionally dependent if they are functions of each other [1, page 214]. Try to express one function through another. Divide the numerator and denominator of g(x, y) by y

$$g(x,y) = \frac{2x - 3y}{4x + 5y} = \frac{\frac{2x}{y} - 3}{\frac{4x}{y} + 5}$$

However

$$\frac{x}{y} = h(x, y)$$

Then we have

$$g(x, y) = \frac{2h(x, y) - 3}{4h(x, y) + 5}$$

Thus the function g(x, y) is the function of h(x, y) and one can say that these functions are functionally dependent.

Answer: functions g(x, y) and h(x, y) are functionally dependent.

References:

[1] S.S. Sastry. Engineering mathematics, volume two, 4th Edition.