Answer on Question #66008 - Math - Calculus

Question

Find the values of a and b for which the following limit exists.
lim x→0 (ae^x +e^-x +bx)/(1-cosx)

$$\lim_{x\to 0}\frac{ae^x + e^{-x} + bx}{1 - \cos x}$$

Solution

Since the denominator $1 - \cos x \to 0$ as $x \to 0$ then limit exists (finite) if numerator $ae^x + e^{-x} + bx \to 0$ as well, so we have

$ae^0 + e^0 + b \cdot 0 = 0 \Longrightarrow a = -1$

Putting $a = -1$ into the original limit we get

$$\lim_{x\to 0}\frac{-e^x + e^{-x} + bx}{1 - \cos x} \qquad (1)$$

Both $-e^x + e^{-x} + bx \to 0$ and $1 - \cos x \to 0$, so we can use L'Hospital's Rule

$$\lim_{x\to 0}\frac{-e^x + e^{-x} + bx}{1 - \cos x} = \lim_{x\to 0}\frac{(-e^x + e^{-x} + bx)'}{(1 - \cos x)'} = \lim_{x\to 0}\frac{-e^x - e^{-x} + b}{\sin x}$$

In order to find b, we repeat the previous steps once again.

Since the denominator $\sin x \to 0$ as $x \to 0$ then the limit exists if the numerator

$-e^x - e^{-x} + b \to 0$ as well, so we have

$-e^0 - e^0 + b = 0 \Longrightarrow b = 2$

Putting $b = 2$ into the (1) we get

$$\lim_{x\to 0}\frac{-e^x - e^{-x} + 2}{\sin x} = \lim_{x\to 0}\frac{(-e^x - e^{-x} + 2)'}{(\sin x)'} = \lim_{x\to 0}\frac{-e^x + e^{-x}}{\cos x} = \frac{-1 + 1}{1} = 0$$

Answer:

The values of a and b for which the limit exists are $a = -1$ and $b = 2$; $\lim_{x\to 0}\frac{-e^x + e^{-x} + 2x}{1 - \cos x} = 0$.