

Answer on Question #66007 – Math – Calculus

Question

True/false. Justify your answer.

$\lim_{(x,y) \rightarrow (0,0)} (x^2 - y^2)/(x^2 + y^2)$ does not exist.

Solution

If the limit exists, then

$$f(x, y) = \frac{x^2 - y^2}{x^2 + y^2}$$

must approach the same limit no matter how (x, y) approaches $(0,0)$ [1, page 918]. Thus, if we can find two different paths of approach along which the function has different limits, then it follows that

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 - y^2}{x^2 + y^2}$$

does not exist. First let's approach $(0,0)$ along the x -axis [1, page 918]. Then $y = 0$ gives

$$f(x, 0) = \frac{x^2}{x^2} = 1$$

for all $x \neq 0$, so $f(x, y) \rightarrow 1$ as $(x, y) \rightarrow (0,0)$ along the x -axis.

We now approach along the y -axis by putting $x = 0$ [1, page 918]. Then

$$f(x, y) = \frac{-y^2}{y^2} = -1$$

for all $y \neq 0$, so $f(x, y) \rightarrow -1$ as $(x, y) \rightarrow (0,0)$ along the y -axis.

Since $f(x, y)$ has two different limits along two different lines, the given limit does not exist.

Answer: True.

References:

[1] James Stewart. Calculus, 7th Edition.