

Answer on Question #66006 – Math – Calculus

Question

True/False. Justify your answer.

The work done by the Force $F(x, y) = (-y, x)$ in moving a particle along the boundary of the ellipse

$$9x^2 + 4y^2 = 36$$

is 6.

Solution

The work done by the Force $F(x, y)$:

$$W = \oint P(x, y)dx + Q(x, y)dy,$$

where

$$P(x, y) = -y ; Q(x, y) = x.$$

The standard equation of the ellipse:

$$\frac{x^2}{2^2} + \frac{y^2}{3^2} = 1$$

The parametric equation of the ellipse:

$$x = 2 \cos t ; y = 3 \sin t ; 0 \leq t \leq 2\pi.$$

Then

$$dx = -2 \sin t dt ; dy = 3 \cos t dt;$$

$$P(x, y)dx + Q(x, y)dy = -ydx + xdy = -3 \sin t (-2 \sin t dt) + 2 \cos t \cdot 3 \cos t dt =$$

$$= 6(\sin t)^2 dt + 6(\cos t)^2 dt = 6dt;$$

$$W = 6 \int_0^{2\pi} dt = 12\pi.$$

Answer: False.

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