Answer on Question #66005 – Math – Calculus

Question

True/False. Justify your answer.

The set $\{(x,\ y,\ z)\ |\ x > 0,\ y > 0,\ z < 0\}$ is a domain in R^3?

Solution

I. Let S denotes this set. Obviously, the set S is an intersection of three different sets S_1, S_2 and S_3:

$$S_1 = \{(x,\ y,\ z)\ |\ x > 0\}$$
$$S_2 = \{(x,\ y,\ z)\ |\ y > 0\}$$
$$S_3 = \{(x,\ y,\ z)\ |\ z < 0\}$$

The set S_1 is open in R^3, since $\forall\ s \in S_1$, we can choose $\varepsilon := x_s/2$ such that the neighbourhood of s, $U_\varepsilon\ =\ \{t = (x_t,\ y_t,\ z_t),\ \ \|t\ -\ s\|\ <\ \varepsilon\} \subset S_1$:

1. $\varepsilon \coloneqq x_s/2 > 0$.

2. $t \in U_\varepsilon \Leftrightarrow |x_t - x_s|^2 + |y_t - y_s|^2 + |z_t - z_s|^2 < e^2 \Longrightarrow x_t = x_s + x_t\ - x_s \geq x_s -\ |x_t - x_s| > x_s - e =\ x_s/2 > 0$.

3. $t \in U_\varepsilon \Longrightarrow\ y_t \in \mathbb{R},\ z_t \in \mathbb{R}$.

Sets S_2 and S_3 are also open. Hence, S is open set as a finite intersection of open sets.

II. Obviously, the set S is path connected: $\forall\ t, s \in S,\ \forall\ \lambda \in [0,\ 1],\ \ p := \lambda t + (1 - \lambda)s \in S$ (since $p_x > 0, p_y > 0, p_z < 0$). It can be shown that every path connected set in R^3 it is also connected.

III. Since the set is open and connected, it is a domain in R^3.

Answer: True. This set is a domain in R^3.