

Answer on Question #66003 – Math – Calculus

Question

True/false. Justify your answer.

The function $f(x, y, z) = e^{xyz}$ is integrable over $[0, 1] \times [0, 1] \times [0, 1]$.

Solution

Let us note that function $g(x) = e^x$ is continuous on $[0, 1]$ (see https://en.wikipedia.org/wiki/Exponential_function). Then function $f(x, y, z) = e^{xyz}$ is continuous on $[0, 1] \times [0, 1] \times [0, 1]$ because e^{abu} is continuous in u on $[0, 1]$ assuming a and b are constants.

Using Riemann-Lebesgue theorem

(see https://en.wikipedia.org/wiki/Riemann_integral#Integrability, it stays true for the multi-dimensional cases) and noting that $e^{xyz} \leq e^{1 \cdot 1 \cdot 1} = e$ on $[0, 1] \times [0, 1] \times [0, 1]$ we conclude that the function $f(x, y, z) = e^{xyz}$ is integrable over $[0, 1] \times [0, 1] \times [0, 1]$.

Answer: True.