Answer on Question #66002 – Math – Calculus

Question

Find the second Taylor polynomial of $f(x, y) = 1 + 3x^2y - 4y^3$ at $(1, 1)$.

Solution

$f(x,y) = 1 + 3x^2y - 4y^3;\ \ f(1,1) = 1 + 3 - 4 = 0;$

$\frac{\partial f}{\partial x} = 6xy; \quad \frac{\partial f(1,1)}{\partial x} = 6;$

$$\frac{\partial f}{\partial y} = 3x^2 - 12y^2; \quad \frac{\partial f(1,1)}{\partial y} = 3 - 12 = -9;$$

$\frac{\partial^2 f}{\partial x^2} = 6y; \quad \frac{\partial^2 f(1,1)}{\partial x^2} = 6;$

$\frac{\partial^2 f}{\partial y^2} = -24y; \quad \frac{\partial^2 f(1,1)}{\partial x^2} = -24;$

$$\frac{\partial^2 f}{\partial x \partial y} = 6x; \quad \frac{\partial^2 f(1,1)}{\partial x \partial y} = 6.$$

The second Taylor polynomial of $f(x, y) = 1 + 3x^2y - 4y^3$ at $(1, 1)$ is

$$T_2(x,y) = f(1,1) + (x-1)\frac{\partial f(1,1)}{\partial x} + (y-1)\frac{\partial f(1,1)}{\partial y} + \frac{1}{2!}\left[(x-1)^2\frac{\partial^2 f(1,1)}{\partial x^2} + 2(x-1)(y-1)\frac{\partial^2 f(1,1)}{\partial x \partial y} + (y-1)^2\frac{\partial^2 f(1,1)}{\partial y^2}\right]$$

$$= 0 + 6(x-1) - 9(y-1) + \frac{1}{2}(6(x-1)^2 + 12(x-1)(y-1) - 24(y-1)^2)$$

$$= 3(x^2 - 2x - 4y^2 + 3y + 2xy)$$

Answer:

$0 + 6(x-1) - 9(y-1) + \frac{1}{2}(6(x-1)^2 + 12(x-1)(y-1) - 24(y-1)^2) =$

$= 3(x^2 - 2x - 4y^2 + 3y + 2xy).$