Answer on Question #66000 – Math – Statistics and Probability

Question

Suppose the gene 'A' stands for tall and gene 'a' stands for short, and 'A' is dominant over
'a'. Suppose the parents are of genotype aa and Aa.
i) In a family like this what is the probability of having a short child?
ii) In a family like this if there are four children, what is the probability that two children are tall and two children are short?
iii) If you survey a large number of such families having four children, what is the average number of tall children to be expected in a family?
iv) If you survey 256 families of this type, each having four children, how many families are expected to have all tall children?

Solution

i) Possible combinations of genes are aA, aa, aA, aa.
So the probability of having a short child is $\frac{2}{4} = \frac{1}{2}$.

ii) Using the binomial probability with $p = 0.5$, $n = 4$, $x = 2$, the probability is
$$P(X = 2) = C_4^2 0.5^2 0.5^2 = \frac{3}{8} = 0.375.$$

iii) $P(X = 0) = P(X = 4) = \frac{1}{16}$, $P(X = 1) = \frac{1}{4}$, $P(X = 2) = \frac{3}{8}$, $P(X = 3) = \frac{1}{4}$
$$E(X) = \frac{1}{16} * 0 + \frac{1}{4} * 1 + \frac{3}{8} * 2 + \frac{1}{4} * 3 + \frac{1}{16} * 4 = 2.$$

iv) $n = N * P(X = 4) = 256 * \frac{1}{16} = 16.$