

Answer on Question #65999 – Math – Calculus

Question

Near what points may the surface

$$x^3 + 3y^2 + 8xz^2 - 3yz^3 = 1$$

be represented as a graph of a differentiable function $z = k(x, y)$? Justify your answer.

Solution

Let $F(x, y, z) = x^3 + 3y^2 + 8xz^2 - 3yz^3 - 1$.

Using the Implicit function theorem, a function $z = k(x, y)$ exists if

$$F'_z(x, y, z) \neq 0$$

Then

$$F'_z(x, y, z) = 16xz - 9yz^2 = z(16x - 9yz) \neq 0,$$

hence a point (x, y, z) meets the conditions $z \neq 0$ and $16x \neq 9yz$.

Answer: $z \neq 0$ and $16x \neq 9yz$.