Answer on Question #65918 – Math – Calculus

Question

Find the volume of the solid bounded by the planes $z = 0, x = 1, x = 2, y = -1,$

$y = 1$ and the surface $z = x^2 + y^2$.

Solution

We shall use the concept of triple integrals (for example see http://tutorial.math.lamar.edu/Classes/CalcIII/TripleIntegrals.aspx).

In our case

$$V = \iiint_E dV = \iiint_E dxdydz = \iint_D \left(\int_0^{x^2+y^2} dz \right) dxdy,$$

where $D = [1; 2] \times [-1; 1]$.

Since

$$\int_0^{x^2+y^2} dz = x^2 + y^2$$

we have

$V = \iint_D (x^2 + y^2) dxdy = \int_1^2 \left(\int_{-1}^1 (x^2 + y^2) dy \right) dx =$

$= \int_1^2 \left(x^2 y + \frac{y^3}{3} |_{y=-1}^{y=1} \right) dx = 2 \int_1^2 \left(x^2 + \frac{1}{3} \right) dx = 2 \left(\frac{x^3}{3} + \frac{x}{3} \right) |_{x=1}^{x=2} = 2 \cdot \left(\frac{8}{3} + \frac{2}{3} - \frac{1}{3} - \frac{1}{3} \right) = \frac{16}{3} =$

$= 5\frac{1}{3}.$

Answer: $5\frac{1}{3}$.