Answer on Question #65898 – Math – Statistics and Probability

Question

In a railway yard, goods trains arrive at the rate of 60 trains per day. Assuming that the inter-arrival time follows an exponential distribution and the service time distribution is also exponential with an average of 72 minutes, calculate the following:

i) The average number of trains in the queue.

ii) The probability that the queue size is greater than or equal to 10.

Solution

We have **M/M/1 queue** (see https://en.wikipedia.org/wiki/M/M/1_queue).

In our case $\lambda = \frac{60\ trains}{24\ hours} = 2.5 \frac{trains}{hour}$; $\mu = \frac{1\ train}{72\ minutes} = \frac{1\ train}{1.2\ hours} = 0.83\ \frac{trains}{hour}$.

Then $\rho = \frac{\lambda}{\mu} \approx 3 > 1$. It means that our queue grows infinitely long and the system will not have a stationary distribution. Then:

i) the average number of trains in system is $\frac{\rho}{1-\rho}$

(see https://en.wikipedia.org/wiki/M/M/1_queue#Number_of_customers_in_the_system) but in our case this value is negative! From another hand, 2.5 trains arrive in the system every hour (in average), and 0.83 trains leave the system every hour (in average). Then the average number of trains in system is equal to

$t \cdot (2.5 - 0.83) = 1.67 \cdot t$ where t is in hours. Then the average number of trains in the queue is $1.67 \cdot t - 1$ because 1 train is in the service and other trains are in the queue. So the limit average number of trains in the queue is $\lim_{t\to\infty}(1.67 \cdot t - 1) = \infty$.

ii) If the queue size is greater than or equal to 10 then the number of trains in a railway yard is greater than or equal to 11 (see i)). The probability that there are i trains in a railway yard is equal to $(1 - \rho)\rho^i$

(see https://en.wikipedia.org/wiki/M/M/1_queue#Number_of_customers_in_the_system), then the required probability is $\sum_{i=11}^{\infty}(1 - \rho)\rho^i = (1 - 3)\sum_{i=11}^{\infty} 3^i = -\infty$.

This answer does not make sense because our system does not have stationary distribution.

Answer:

i) ∞; **ii)** there is no answer.